

# Assessment of a Multiperiod Optimal Power Flow for Power System Operation

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**Abstract** – The optimal power flow is an important tool for power system planning and power system operation. It is used in a 24-hour period to find an economic dispatch of generating units considering network restrictions. The optimal power flow provides valuable information about the operation cost, the transmission flows, the generation and the congestion in the system. This information is used by generators, planners, operators and regulators in order to analyze and take decisions about the system at short and long term. The first one corresponds to the information for the operation. The second one corresponds to the information for the planning. This paper proposes a detailed optimal power flow formulation looking for a minimum cost of generation considering wind generation. Five solvers (CBC, CLP, CPLEX, Gurobi and GLPK.) have been used in order to compare differences between them. These solvers are commonly used to solve the multiperiod DC optimal power flow. An IEEE-24 test system is used to compare the solutions provided by the solvers. The findings reveal significant differences between the solvers when they are used to solve the IEEE-24 test system. Additionally, the computing time for each solver is reported. The solvers CPLEX and Gurobi show the lowest computational time to find a solution.

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**Keywords:** Generation, Optimal Power Flow, Wind power, Optimization, Power Systems, Renewable Energy

## Nomenclature

$g$	Thermal generation units	$PL_{ij,t}$	Active power flow of branch connecting bus $i$ to $j$ at time $t$ [MW]
$i,j$	Index of network buses connected by transmission branches	$P_{i,t}^{Gen}$	Active power generated by thermal unit $g$ at time $t$ [MW]
$t$	Time period [h]	$P_{i,t}^{wind}$	Active power generated by wind turbine connected to bus $i$ at time $t$ [MW]
$G$	Number of thermal generation units	$P_{i,t}^{wl}$	Curtailed power of wind turbine connected to bus $i$ at time $t$ [MW]
$L$	Number of network branches	$\lambda_{i,t}$	Dual variable: Locational Marginal Price in bus $i$ at time $t$ [\$/MWh]
$T$	Time period in the planning horizon (24 hours)	$F_{obj}$	24-hour Total operating costs [\$/MWh]
$N$	Number of network buses	$\theta_{i,t}$	Voltage angle of bus $i$ at time $t$ (rad)
$VWL$	Value of loss of wind [\$/MWh]	$P_{i,t}^{wl}$	Curtailed power of wind turbine connected to bus $i$ at time $t$ [MW]
$X_{ij}$	Reactance of branch connecting bus $i$ to $j$	DCOPF	DC Optimal Power Flow
$b_g$	Fuel cost coefficients of thermal unit $g$	JuMP	Julia for Mathematical Optimization
$P_g^{\min}$	Minimum limits of power generation of thermal unit $g$	LMP	Locational Marginal Price
$P_g^{\max}$	Maximum limits of power generation of thermal unit $g$	LP	Linear Programming
$PL_{ij}^{\max}$	Maximum power flow limits of branch connecting bus $i$ to $j$	RTS	Reliability Test System
$D_{i,t}$	Electric power demand in bus $i$ at time $t$		
$Av_t^{wind}$	Availability of wind turbine connected to bus $i$ at time $t$ [MW]		
$C_i^{wind}$	Capacity of wind turbine connected to bus $i$ [MW]		
$R_g^{up}$	Ramp-up limit of thermal generation unit $g$ [MW/h]		
$R_g^{down}$	Ramp-down limit of thermal generation unit $g$ [MW/h]		

## I. Introduction

The planning of power systems operation represents a complex challenge today given the integration of renewables and storage systems. The planning stage includes a multiperiod analysis in order to schedule generation resources according to the transmission system and a required demand in a period. The Optimal

Power Flow (OPF) constitutes the core for power system operation since it represents the tool for decision-making in the planning and dispatching of generation resources under network constrained. The optimal power flow problem is intrinsically difficult due to non-convexities and the setting of multi-part nonlinear pricing functions.

Given the mathematical complexity in the OPF formulation, a linearized version can be obtained under some operation considerations. Some assumptions are made about considering only active power. This linearized version is well known as DCOPF. The current research is active working on different approaches and methods to solve the OPF with different formulations, algorithms and technological resources. For instance, the authors in [1] have proposed a multi-objective approach to solve an optimal approach for power system expansion planning considering distributed generation. In [2] an OPF is determined with an interline power flow controller to integrate FACTS (Flexible AC transmission system). The authors in [3] have proposed an optimization method in order to determine an optimal configuration of a TCSC (Thyristor-Controlled Series Capacitor) controller. Under the integration of renewable resources and storage systems the power systems operation represents a challenge. Different approaches have been explored and multiple solvers have been used to solve the optimization problem. For instance, wind generation modeling in the OPF problem has been included in [4]-[7]. The integration of wind power to electrical systems creates new challenges because the wind power is intrinsically intermittent. Therefore, there are different approaches to model wind power under reliability requirements. For instance, in [8] the authors have proposed a forecasting method in order to deal with wind power variability. A model predictive control has been proposed as an algorithm for wind energy conversion in [9]. In [10], a wind curtailment method has been proposed. In others proposals other features such as branches and generation constraints that can occur in a certain period have been included in [11] and [12]. Other authors have made comparisons and analyses between this approach and the conventional methods without these variables [13]. On the other hand, this optimization problem has also been raised from the Energy Storage Systems integration (ESS), because it is an attractive option to increase the flexibility for operation and planning of power systems [14]. These units can absorb energy in case of excessive generation or low electricity prices, mitigating the problem of uncertainty in the renewable sources. Several studies like [15] have researched about this integration to the economic dispatch using multiperiod OPF due to present specific challenges to the traditional OPF paradigm such as the modeling of charge/discharge dynamics of ESS systems. Other studies have included more variables in order to bring the problem closer to a more realistic context such as [16] using energy loss constraints on the transmission branches to evaluate different generation scenarios. On the other hand, [17] has added an

environmental approach, using variables such as emission generation in order to optimize the total production costs, using the thermal generation as little as possible, without neglecting the reliability in the system.

There are different approaches in the literature of optimization methods in order to solve the issues related to electrical systems. For instance, in [18], two optimization algorithms (chaotic optimization and a genetic algorithm) have been proposed to tune static synchronous series compensators. Given the complexity with the integration of renewables at distribution level, the authors in [19] have proposed two heuristics algorithms (vortex search algorithm and artificial bee algorithm) in order to solve the multiobjective optimization problems together with new performance indices. They have performed a test using the IEEE-34-bus system. In [20], the authors have proposed a power management system based on an optimization formulation for electrical supply in rural areas and small islands. The results show the effectiveness of the optimal approach. A multi-objective optimization approach has been proposed. In [21] in order to reduce losses in the distribution system and to reduce drops in the voltage ranges. The algorithms are tested on an IEEE 33-bus network with a genetic algorithm with successful results.

In order to design control strategies for power management of EV charging stations that use renewable energy sources an optimization algorithm is defined and the simulations in MATLAB shows the effectiveness for various operating modes [22]. A quadratic optimization approach has been presented in in order to identify vulnerabilities in power system networks [23], [24].

The integration of new technologies under the uncertainty of renewable resources requires optimization approaches to deal with the variability. The authors in [25] have proposed an approach to design a controller for solid state transformers in order to improve transient stability under the operation of solar farms. The results are tested with the IEEE-39 test system. The results and the simulations performed show that the power fluctuation is mitigated with the proposed optimal controller. An optimization algorithm has been developed to control variations in wind speed and load with the use of compensators. Specifically, the authors have proposed a Cuckoo search algorithm and the application in a multi-level scheme [26]. Similarly, in [27], the authors have proposed a genetic algorithm in order to deal with the non-linear and non-convex characteristics for power system planning applied to micro grids. The OPF problem can be even complex if the generator state is included as an integer variable indicating if the unit is on or if the unit is off. This problem has been addressed by the authors in [28] formulating a modified Lagrange multiplier. This formulation includes spinning reserve and start-up costs. Additionally, an approach using optimization methods has been proposed in [29] in order to assure feasibility autonomous buildings with electricity by renewables. A different approach for the optimal power flow problem has been presented in [30]

using a novel sine cosine optimization algorithm. This work presents a full description of Pareto optimal solutions. The effectiveness is validated using two systems. On the other hand, in electrical distribution systems there are some challenges in the operation that could be handled with an optimal approach. For instance, a particle swarm optimization algorithm has been proposed in [31] in order to improve the performance in terms of losses, voltage profile and loading balance. The approach is validated in two IEEE cases with a radial network topology. The research about approaches to improve the performance in radial distribution networks has various perspectives. Usually, in distribution networks, the voltage profile is close to operational limits. Sometimes, the problem is related with low availability of reactive power. In those cases, the capacitors represent a solution to provide reactive power.

Precisely, in [32], the authors have proposed a heuristic approach using an artificial bee colony in order to solve the numerical problem about the allocation and the size of optimal shunt capacitors. The approach is tested in two systems, with 41 and 69 nodes, with satisfactory results. Nowadays, along the integration of renewable resources there are other resources such as the demand response. The demand response corresponds to offers from users who bid a decrease in the power demand. In terms of planning and operation, this new resource imposes new challenges for the power dispatching. A novel work has been represented in [33]; in this paper the authors have developed a framework based on an OPF formulation to deal simultaneously with renewables resources and demand response. An optimization model for integrated generation and transmission planning has been proposed in [34] and [35]. These papers include a consideration about risk related with the intermittency of renewables. Additionally, these papers take in to account high-level of hydroelectric generation. About transmission expansion planning, the authors in [36] have proposed a heuristic algorithm based on particle swarm optimization in order to decide about new transmission lines. This paper provides an assessment of a multiperiod OPF using five different solvers in order to solve the optimization problem and illustrating about the differences between them in terms of generations patterns. The results highlight the differences between then for the IEEE 24-bus test system.

The paper is organized as follows. The problem formulation is presented in Section II. In Section III, the proposed procedure is tested and it is analyzed using the IEEE 24-bus modified test system. Section IV provides some concluding remarks.

## II. Multiperiod Optimal Power Flow Formulation

This section includes the complete formulation to assess a multiperiod optimal power flow for power system operation. The complete set of variables,

parameters and indices is described as follows. The formulation is expressed as Linear Programming (LP) optimization problem to address a minimum total operating cost associated with producing with thermal and wind energy (including wind curtailment dynamics) sources for a 24-hour period described by (1). Equation (2) indicates the total cost of energy production with thermal units during an interval of time  $T$  and (3) refers to the production costs associated with not taking full advantage of the source of wind generation available during this same interval of time:

$$F_{obj} = C_G + W_L \quad (1)$$

$$C_G = \sum_{t=1}^T \sum_{g=1}^G b_g P_g^{Gen} \quad (2)$$

$$W_L = \sum_{t=1}^T \sum_{i=1}^N VWL \times P_{i,t}^{wl} \quad (3)$$

The restrictions for the dispatching model are given by the power flow equations. This paper uses the DC model to include power flow calculations. The power flow balance is given by equation (4). The power flowing on each line is given by (5). The power flow restrictions are given by the boundaries in (6):

$$\sum_{g=1}^G P_{g,t}^{gen} + P_{i,t}^{wind} - D_{i,t} = \sum_{j=1}^L PL_{ij,t} \quad (4)$$

$$PL_{ij,t} = \frac{1}{X_{ij}} (\theta_{i,t} - \theta_{j,t}) \quad (5)$$

$$-PL_{ij}^{\max} \leq PL_{ij,t} \leq PL_{ij}^{\max} \quad (6)$$

The dual variable associated to equation (4) corresponds to the locational marginal price (LMP) of each bus hourly. On the other hand, the restrictions governing the behavior of thermal generation units are defined in (7), (8), and (9), where equation (7) corresponds to the operational range of thermal generators. Equations (8) and (9) indicate the maximum up and down ramps limits that each generator can perform from one hour to the next:

$$P_g^{\min} \leq P_{g,t}^{gen} \leq P_g^{\max} \quad (7)$$

$$P_{g,t}^{Gen} - P_{g,t-1}^{Gen} \leq R_g^{up} \quad (8)$$

$$P_{g,t-1}^{Gen} - P_{g,t}^{Gen} \leq R_g^{down} \quad (9)$$

Finally, the restrictions that determine the behavior of wind generation are defined in (10). This expression

corresponds to the reduction of potentially available wind energy, depending on the capacity of the generator and the amount of wind available in a certain time. Equation (11) describes the minimum and the maximum power range:

$$P_{i,t}^{wl} = Av_t^{wind} C_i^{wind} - P_{i,t}^{wind} \quad (10)$$

$$0 \leq P_{i,t}^{wind} \leq Av_t^{wind} C_i^{wind} \quad (11)$$

### III. Assessment of OPF with Multiple Solvers

The multiperiod OPF economic dispatch framework proposed as a LP optimization problem is assessed using five solvers commonly used: CBC, CLP, CPLEX, Gurobi and GLPK. Each one has been used to solve an OPF case in an IEEE test system. A short description of each solver is provided below. CBC or The COIN Branch and Cut solver in its version 2.9.9 [37] is an open source solver written in C++, intended to be used as a core or library for custom branch and cut solvers. This solver is specialized in problems that can be solved with Mixed Integer Programming (MIP). CLP or Coin-or Linear Programming in its version 1.17 [38], is also an open-source solver written in C++ used as a core or a library, but includes a standalone executable version. On the other hand, this solver is specialized in problems that can be solved with Linear Programming (LP). CPLEX solver or optimizer in its version 12.9.0 [39], is developed by ILOG and acquired by IBM, with commercial and educational licenses implemented in C. It uses robust algorithms for solving LP optimization problems (from short to very large) at high speed, using primal or dual variants as simplex, interior point method and others. It supports many programming and modeling languages.

GUROBI optimizer, in its version 8.1.1 [40], is a commercial solver for different types of programming as Linear Programming (LP), Quadratic Programming (QP), Mixed-Integer Linear Programming (MILP), and others. In addition, this solver uses robust algorithms as CPLEX for solving optimization problems, so their speed, versatility and precision are very good. In the same way, it supports a variety of modeling and programming languages as Java, Python, C++ and others. GLPK or GNU Linear Programming Kit solver in its version 4.32 [41], is an open-source solver written in ANSI C, intended for solving large scale problems as LP, MIP and others, because it handles simplex methods and dual variables, and other methods incorporated in the previous solvers such as cut and branch, among others. It should be remembered that all these solvers are under the Julia language on its JuMP platform 0.19.2 [42].

### IV. Results and Analysis

The IEEE 24-bus Reliability Test System (RTS) is examined in this section in order to assess the

multiperiod OPF. The simulations have been completed by a PC with Intel Core I5+ 8300H @ 2.3 GHz with 12.00 GB RAM. The IEEE 24-bus test system includes 12 thermal units and 3 wind generators. The data are listed in Tables I and II respectively. The cost, ramps, the limits and the bus location are equal to the report in [14].

Table III lists the characteristics such as reactance, rating (power constraints) and related buses of each branch of the system according to [14]. The system daily load curve and the wind availability pattern for the IEEE 24-bus system are plotted in Fig. 1. This figure represents a 24-hour load behavior with increasing tendency as the time advances towards the hours of greater load (i.e. hours 16 to 19). It can be seen that the peaks of both patterns do not coincide (i.e. the maximum wind availability in the hour 15 coincides with a low point in the load curve). The same happens with the load peak (hour 18), which coincides with a decrease in wind availability, which has a greater difficulty in minimizing total operating costs. Both curves are normalized (from 0 to 1) with respect to their maximum values.

TABLE I  
THERMAL UNITS DATA FOR THE IEEE 24-BUS SYSTEM

Gen. #	Bus	$P_g^{\min}$ (MW)	$P_g^{\max}$ (MW)	Marg. Cost (\$)	$R_g^{up}$ (MW/h)	$R_g^{down}$ (MW/h)
1	18	100	400	5.47	47	47
2	21	100	400	5.47	47	47
3	1	30.4	152	13.32	14	14
4	2	30.4	152	13.32	14	14
5	15	54.25	155	16	21	21
6	16	54.25	155	10.52	21	21
7	23	108.5	310	10.52	21	21
8	23	140	350	10.89	28	28
9	7	75	350	20.7	49	49
10	13	206.8	591	20.93	21	21
11	15	12	60	26.11	7	7
12	22	0	300	0	35	35

TABLE II  
WIND GENERATOR DATA FOR THE IEEE 24-BUS SYSTEM

Gen. #	Bus	$C_i^{wind}$
1	8	200
2	19	150
3	21	100

TABLE III  
BRANCH DATA FOR THE IEEE 24-BUS SYSTEM

From	To	$X_{ij}$ (p.u)	Rat	From	To	$X_{ij}$ (p.u)	Rat
1	2	0.0139	175	11	13	0.0476	500
1	3	0.2112	175	11	14	0.0418	500
1	5	0.0845	175	12	13	0.0476	500
2	4	0.1267	175	12	23	0.0966	500
2	6	0.1920	175	13	23	0.0865	500
3	9	0.1190	175	14	16	0.0389	500
3	24	0.0839	400	15	16	0.0173	500
4	9	0.1037	175	15	21	0.0245	1000
5	10	0.0833	175	15	24	0.0519	500
6	10	0.0605	175	16	17	0.0259	500
7	8	0.0614	175	16	19	0.0231	500
8	9	0.1651	175	17	18	0.0144	500
8	10	0.1651	175	17	22	0.1053	500
9	11	0.0839	400	18	21	0.0130	1000
9	12	0.0839	400	19	20	0.0198	1000
10	11	0.0839	400	20	23	0.0108	1000
10	12	0.0839	400	21	22	0.0678	500

Rat = Rating in MVA unit

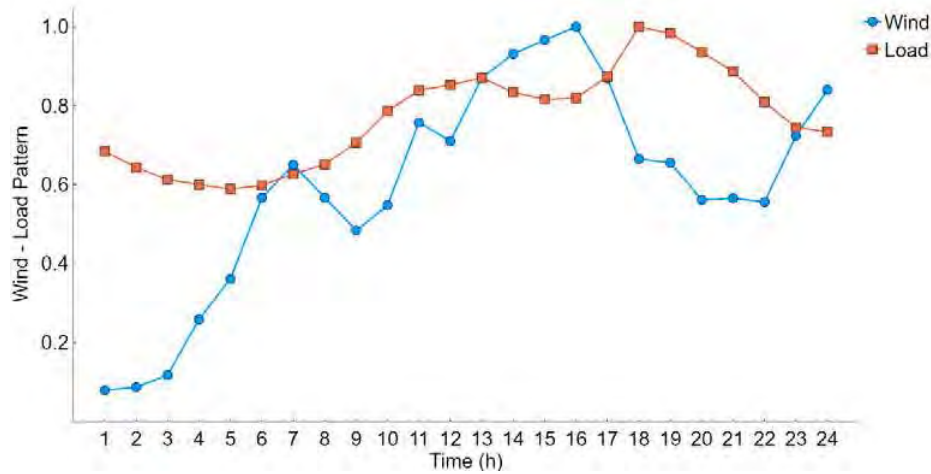


Fig. 1. Wind and load variation patterns during 24-hour horizon

For two scenarios, this paper assesses the difference between the five solvers, performing the LP optimization problem regarding the scheduling of thermal units in a 24-hour period. The first scenario uses the parameters of the IEEE 24-bus system described in [14]. In this case, the generation parameters are presented as minimum and maximum capacities, and the marginal cost in two pairs of thermal generators as evidenced in Table I. This scenario explores an important condition where there are two or more thermal units with the same availability and conditions to deliver power to the system, considering the existence of multiple scheduling solutions in the thermal generation given the approximations of the solvers used, situation that is desired to evaluate. The second scenario uses the parameters of the system as the previous one. However, it makes the modification of a crucial parameter. It consists in the increase of the marginal cost of one of the thermal units that present similar information in 30% for both cases where it is presented (i.e. generators 1 and 3). This scenario explores the similarity of information between the generators where it occurs and evaluates the agreement between the scheduling results of thermal generation. That means that these kinds of similarities can turn this optimization problem into one where more than one solution can be found.

#### IV.1. Results: Scenario 1

When tests with different solvers are performed, it has been shown that in all the cases the same total operating cost is obtained (objective of the primal optimization problem that describes the entire system) as shown in Table IV. The power production scheduling values obtained in the scenario 1 for thermal units mentioned above according to each solver are not equivalent between them as shown in Figs. 2. It should be clarified that the other generation units do not exhibit this behavior and the generation quantities match in each of the cases.

In hours 2 to 7 and 13 to 16, it can be seen that the

different solvers have performed the optimization processes with the same information. Generators 1 and 2 have different generation levels, which means that this optimization problem can have more than one solution, despite as shown in Table IV where the total operating cost is the same in all cases.

#### IV.2. Results: Scenario 2

In scenario 2, the generation cost has been increased by 30% in cases where thermal units have repeated generation data (in this case units 1, 2, 3 and 4), in order to check the sensitivity of this variable to the LP optimization problem and to verify that the solvers determine different generation schedules for this type of plants, as well as the respective wind power generation.

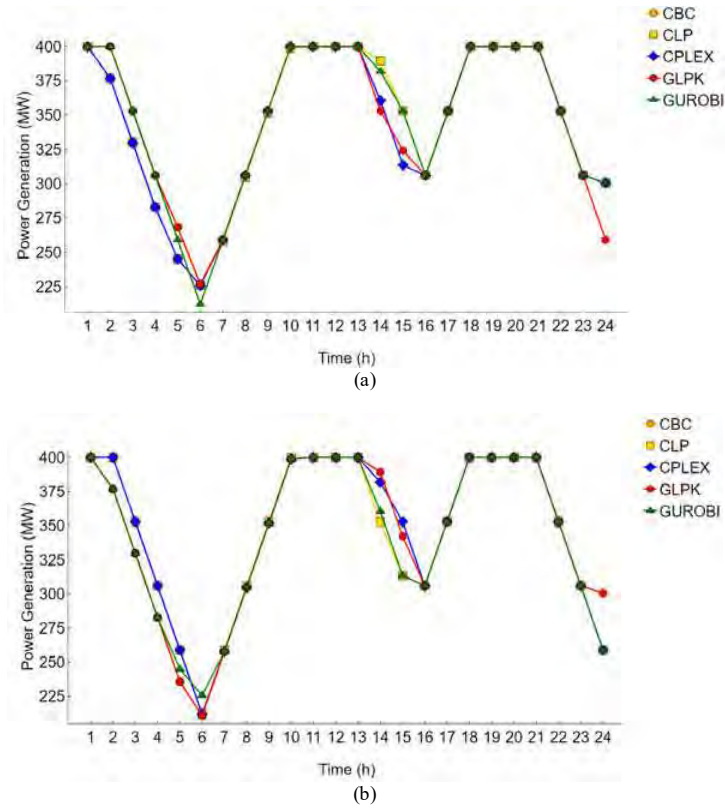
The power production scheduling values obtained in the scenario 2 for thermal units mentioned above according to each solver are not equivalent between them as shown in Figs. 2. In this scenario the solvers solution is shown in Table V. However, as shown in Figs. 3, the generation schedule in the generators has changed in comparison with the scenario 1.

TABLE IV  
TOTAL OPERATING COST RESULTS OBTAINED FOR EACH SOLVER  
(BASE CASE)

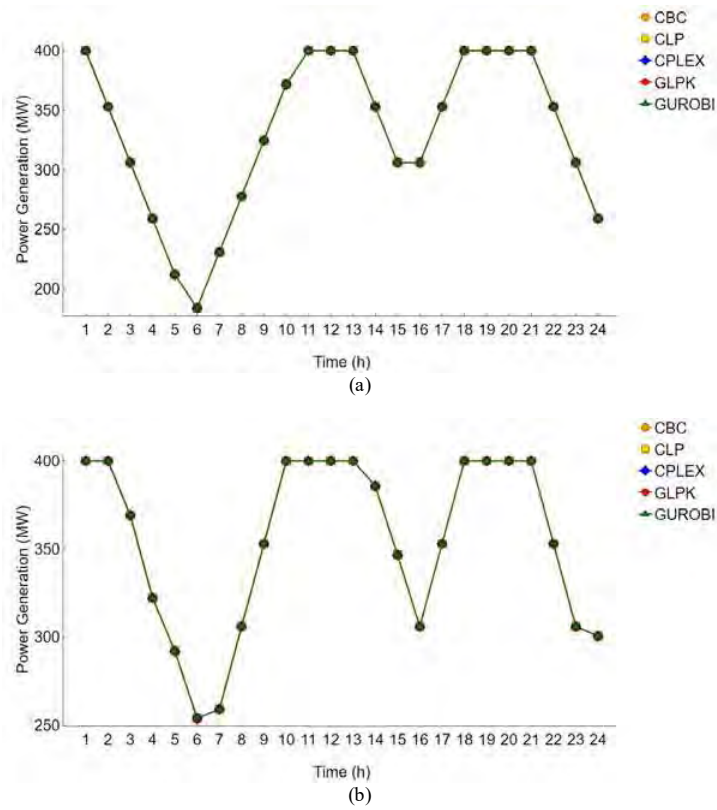
Solver	Total Operating Cost (\$)
CBC	432292.56
CLP	432292.56
CPLEX	432292.56
Gurobi	432292.56
GLPK	432292.56

TABLE V  
TOTAL OPERATING COST RESULTS OBTAINED FOR EACH SOLVER  
(GENERATION COST MODIFIED CASE)

Solver	Total Operating Cost (\$)
CBC	449109.18
CLP	449109.18
CPLEX	449109.18
Gurobi	449109.18
GLPK	449109.18



Figs. 2. Comparison of thermal power scheduling between solvers for generators  $G_1$  (a) and  $G_2$  (b) (Scenario 1)



Figs. 3. Comparison of thermal power scheduling between solvers for generators  $G_1$  (a) and  $G_2$  (b) (Scenario 2)

### IV.3. Assessment of Solvers

In addition to comparing the results of the generation schedules, speed tests have been performed with each solver in order to see which one of them could solve this problem of optimization more quickly, as it can be seen in Table VI. The solvers CPLEX and Gurobi are the ones that have taken less time. It can be seen that among the proposed solvers, CPLEX and Gurobi have the lowest CPU time (0.02 s), since they have a more robust development and support than the other solvers raised.

This explains why these solvers are more frequently used for solving problems of optimization of this type and more desirable in power systems with a greater number of buses.

In order to check what could be seen visually in the Figs. 2 and Figs. 3, the solution provided by the solver Gurobi has been taken as a reference with respect to the hourly programming of the thermal units. The different solvers have been compared in their hourly results through the error with respect to these reference values as shown in Table VII and Table VIII.

In both tables, it can be noticed that there are divergences in the results in all the solvers referred to above in hours 5, 6 and 14. In these same points in the previous plots, this same difference is appreciated.

However, these are not the only errors that have occurred with the solvers used.

TABLE VI  
WIND GENERATOR DATA FOR THE IEEE 24-BUS SYSTEM

Solver	Total CPU time(s)
CBC	0.080
CLP	0.052
CPLEX	0.020
Gurobi	0.020
GLPK	0.032

TABLE VII  
ERRORS IN THE THERMAL GENERATORS SCHEDULING OF THE  
DIFFERENT SOLVERS, TAKING AS REFERENCE THE GUROBI SOLVER –  
GEN 1

T	Gurobi (MW)	CBC (%)	CLP (%)	CPLEX (%)	GLPK (%)
1	400.00	0.0	0.0	0.0	0.0
2	400.00	0.0	5.80	5.80	0.0
3	353.00	0.0	6.57	6.57	0.0
4	306.00	0.0	7.58	7.58	0.0
5	259.00	3.57	5.39	5.39	3.57
6	212.00	6.99	6.46	6.46	6.99
7	259.00	0.0	0.43	0.0	0.0
8	306.00	0.0	0.36	0.0	0.0
9	353.00	0.0	0.31	0.0	0.0
10	400.00	0.0	0.28	0.0	0.0
11	400.00	0.0	0.0	0.0	0.0
12	400.00	0.0	0.0	0.0	0.0
13	400.00	0.0	0.0	0.0	0.0
14	381.85	1.97	1.97	5.58	7.55
15	353.00	0.0	0.0	11.18	8.17
16	306.00	0.0	0.0	0.0	0.0
17	353.00	0.0	0.0	0.0	0.0
18	400.00	0.0	0.0	0.0	0.0
19	400.00	0.0	0.0	0.0	0.0
20	400.00	0.0	0.0	0.0	0.0
21	400.00	0.0	0.0	0.0	0.0
22	353.00	0.0	0.0	0.0	0.0
23	306.00	0.0	0.0	0.0	0.0
24	300.51	0.0	0.0	0.0	13.81

TABLE VIII  
ERRORS IN THE THERMAL GENERATORS SCHEDULING OF THE  
DIFFERENT SOLVERS, TAKING AS REFERENCE THE GUROBI SOLVER –  
GEN 2

T	Gurobi (MW)	CBC (%)	CLP (%)	CPLEX (%)	GLPK (%)
1	400.00	0.0	0.0	0.0	0.0
2	376.79	0.0	6.15	6.15	0.0
3	329.79	0.0	7.03	7.03	0.0
4	282.79	0.0	8.20	8.20	0.0
5	245.04	3.77	5.70	5.70	3.77
6	225.70	6.56	6.07	6.07	6.56
7	257.89	0.0	0.43	0.0	0.0
8	304.89	0.0	0.36	0.0	0.0
9	351.89	0.0	0.31	0.0	0.0
10	398.89	0.0	0.28	0.0	0.0
11	400.00	0.0	0.0	0.0	0.0
12	400.00	0.0	0.0	0.0	0.0
13	400.00	0.0	0.0	5.91	0.0
14	360.52	2.09	2.09	12.59	8.00
15	313.52	0.0	0.0	0.0	9.20
16	306.00	0.0	0.0	0.0	0.0
17	353.00	0.0	0.0	0.0	0.0
18	400.00	0.0	0.0	0.0	0.0
19	400.00	0.0	0.0	0.0	0.0
20	400.00	0.0	0.0	0.0	0.0
21	400.00	0.0	0.0	0.0	0.0
22	353.00	0.0	0.0	0.0	0.0
23	306.00	0.0	0.0	0.0	0.0
24	300.51	0.0	0.0	0.0	13.81

## V. Conclusion

This paper presents an assessment of solutions provided by various solvers commonly used to solve the optimal power flow problem with applications to operation and planning. This paper presents a detailed comparison between optimization solvers with some important findings. The most important finding is the divergence between solutions provided by optimization solvers. With an IEEE case, this paper shows different solutions with the solvers in critical dispatching points.

This means that the optimizer plays a critical role to find solutions even in a linear problem. Usually, the results in papers do not discuss about the quality of the solution. One of the findings has been to disclosure the solvers CLP and CPLEX delivers a different solution provided by Gurobi for two generators in the power system with critical implications in the solution. CPLEX and Gurobi are commercial software and CLP is open source. The proposed methodology can be used for different power system cases with operational and planning purposes. In order to verify cases with multiple solutions in the optimal dispatching for a 24-hour period.

Moreover, the proposed methodology can be extended to other solvers and to find cases with multiple solutions.

An important and critical research direction would be including the power equations in the nonlinear version in order to consider reactive power and other variables.

## Acknowledgements

The authors would like to thank to the Universidad Autónoma de Occidente, Cali, Colombia for its financial support.

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